Reliable output feedback control for systems with control input delay and stochastic actuator failure

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***Abstract*— This paper is concerned with the design of reliable static output feedback controllers (RSOFC) for systems with stochastic actuators failure. A sufficient condition for the existence of reliable RSOFC is presented in terms of a set of linear matrix inequalities (LMIs), which guarantees that the closed-loop system, with consideration of actuators failure, satisfies a desired H_∞ disturbance attenuation γ. A numerical example is presented to illustrate the effectiveness of the proposed design method.**

***Index Terms*— Reliable Static Output Feedback, Control Input, Time Delay**

I. INTRODUCTION

Over the past few decades, time-delay systems have drawn much attention from researchers throughout the world. This is due to their important role in many practical systems. A great number of research results concerning time-delay systems exist in the literature[1], [2], [3]. Many early theoretical developments assume that all the system states are accessible. However in the case where only a subset of states are measurable, which is relevant to a range of practical applications, static output feedback control is more realistic.

In recent years, the study of reliable control has received considerable attention due to the growing demands on reliability. The main task of this study is to design a fixed controller such that the closed-loop system can maintain stability and performance, not only when all control components are operational, but also in case of some admissible control component outages. Reliable control problems for linear systems have been extensively studied [4], [5], [6], [7], [8], [9], [10]. Most of them depicte the failure model by introducing a scaling factor, such as, defining $\beta_l \in \Omega \triangleq \{\beta_l = diag[\beta_{l_1} \quad \beta_{l_2}, \cdots, \beta_{l_q}], \beta_{l_i} = 0 \ or \ 1, i = 1, 2, \cdots, q\}$, that is, when a failure occurs, the control action simply becomes zero, in fact, the actuator may not be completely failure, that is, the scale factor $\beta_{li} = 0$ or 1 are only two special cases. By decomposing the control matrix B into B_Σ and $B_{\bar{\Sigma}}$ is the other type failure modeling method[11], [12], where B_Σ denotes the control matrix associated with the set Σ and $B_{\bar{\Sigma}}$denotes the control matrix associated with the complementary subset of the control input, and B_σ with $\sigma \subseteq \Sigma$ correspond to a subset of susceptible actuator experience failure. However, it cannot represent actuator fault exactly. In practical systems, because of actuators aging, zero shift, electromagnetic interference, nonlinear amplification in different frequency field, and so on. It will be more reasonable if the fault scale factor obeys a certain probabilistic distribution in an interval. To the best of our knowledge, it seems that there are few results on the problem of reliable control with such a actuator fault model which satisfies a certain probabilistic distribution, which is not only theoretically interesting and challenging, but also very important in practical applications. This greatly motivates the study of this work.

In this paper, A more general actuator fault model is proposed, which satisfies a certain probabilistic distribution in an interval. we are interesting in designing a reliable output feedback controller such that the dynamic system is exponentially mean-square stable despite possible actuator signals drift or missing.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of real $n \times m$ matrices, I is the identity matrix of appropriate dimensions, $\|\cdot\|$ stands for the Euclidean vector norm or spectral norm as appropriate. The notation $X > 0$ (respectively, $X < 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, negative definite). When x is a stochastic variable , $\mathscr{E}\{x\}$ stands for the expectation of x. The asterisk $*$ in a matrix is used to denote term that is induced by symmetry, Matrices, if they are not explicitly stated , are assumed to have compatible dimensions.

II. PROBLEM FORMULATION

Consider the following time-delay system:

$$\dot{x}(t) = Ax(t) + B_1u(t) + B_2\omega(t) \tag{1a}$$
$$y(t) = Cx(t) \tag{1b}$$
$$z(t) = C_2x(t) + D_2\omega(t) \tag{1c}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^l$ is the control input, $\omega(t)$ is the disturbance input which belongs to $L_2[0, \infty)$, and $z(t)$ is the controlled output. A, B_1, B_2, C, C_2 and D_2 are known constant matrices, and supposing C is of full rank.

We consider the following static state feedback controller for the system (1a)

$$u(t) = Ky(t - \tau(t)) \tag{2}$$

where K is a feedback matrix to be determined, $\tau(t)$ denotes the control input delay and satisfies $\tau_1 \le \tau(t) \le \tau_2$.

Let $u^F(t)$ represent the control input after actuator failures occurred, Then the following fault model is adopted for this

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study

$$\begin{aligned} u^F(t) &= \Xi u(t) \\ &= \sum_{i=1}^{m} \xi_i L_i KCx(t-\tau(t)) \end{aligned} \quad (3)$$

where $\Xi = diag\{\xi_1 \cdots \xi_m\}$ with $\xi_i (i = 1, \cdots, m)$ are m unrelated random variables, and the mathematical expectation and variance of ξ_i is assumed as μ_i and σ_i^2 respectively, $L_i = diag\{\underbrace{0, \cdots, 0}_{i-1}, 1, \underbrace{0, \cdots, 0}_{m-i}\}$. For convenience, we also define $\bar{\Xi} = diag\{\mu_1, \cdots, \mu_m\}$ and $\Delta = diag\{\sigma_1, \cdots, \sigma_m\}$.

Remark 1 By introducing a a random matrix Ξ, satisfying a certain probabilistic distribution in an interval, a phenomenon of actuator failure is then presented. Specially, for $\xi_i = 0$, it means complete failure of the ith actuator; for $\xi_i = 1$, it means the ith actuator is in good working condition; for $0 < \xi_i < 1$, it means partial failure of the ith actuator ; for $\xi_i > 1$, it means the actuator-amplifier with forward drift. It should be noted that, in many cases, the gain of actuators could be larger than normal cases by reasons of the surrounding influence or actuator-amplifiers themselves. Therefore, the mathematical expectation μ_i of random variance ξ_i, similar to the scaling factor in [13], should be defined as $0 < \mu_i < \bar{\mu}_i$, where $\bar{\mu} \geq 1$. Furthermore, σ_i denotes the gain of actuators fluctuation levels because of influence of all the factors acting on actuators.

Remark 2 $\mu_i = \mathcal{E}\{\xi_i\}$ represents for the failure rate of the ith actuator. It should be noted that with the consideration of the influence of all the factors, $\mu_i = 1$ does not mean the ith actuator always in good working condition, the values of ζ_i can be bigger or smaller than 1. Similarly, $\mu_i = 0$ does not mean the complete failure of the ith actuator. In particular, If the case $\mu_i = 0$, and $\sigma_i = 0$, simultaneity, it stands for an entire missing of signals, and if $\mu_i = 1$, $\sigma_i = 0$ indicates intactness. In fact, actuator-amplifiers backward or forward drift usually occurs in practice situations, while completely failure and intactness are only two special cases.

Combining (1a) and (3), we obtain the following close-loop system as follows

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1\bar{\Xi}KCx(t-\tau(t)) \\ &+ B_1(\Xi - \bar{\Xi})KCx(t-\tau(t)) + B_2\omega(t) \end{aligned} \quad (4)$$

The objective of this study is to develop a reliable controller for the closed-loop system with stochastic fault model described by (4). For this purpose, the following Lemma derived from Jessen's inequality and definitions are introduced.

Lemma 1[14] For any constant matrix $T \in \mathbb{R}^{n\times n}$, $T > 0$, scalars $\tau_1 \leq \tau(t) \leq \tau_2$, and vector function $\dot{x} : [-\tau_1, 0] \to \mathbb{R}^n$ such that the following integration is well defined, it holds that

$$-(\tau_2 - \tau_1)\int_{t-\tau_2}^{t-\tau_1} \dot{x}(t)T\dot{x}(t) \leq$$
$$\begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \\ x(t-\tau_2) \end{bmatrix}^T \begin{bmatrix} -T & T & 0 \\ * & -2T & T \\ * & * & -T \end{bmatrix} \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \\ x(t-\tau_2) \end{bmatrix}$$

Definition 1 For a given function $V : C^b_{F_0}([-\tau_2, 0], R^n) \times S$, its infinitesimal operator $\mathcal{L}$[15] is defined as

$$\mathcal{L}V(x_t) = \lim_{\Delta\to 0^+} \frac{1}{\Delta}[\mathscr{E}(V(x_{t+\Delta}|x_t) - V(x_t))] \quad (6)$$

III. MAIN RESULT

In this section, we aim to develop an innovative approach to guarantee the system (4) is exponentially mean-square stable. The controller K could be solved from the following results.

Theorem 1 For given scalars $\tau_1, \tau_2, \sigma_i, \mu_i (i = 1, \cdots, m)$ and matrix K, the system (1) with sensor distortion (3) is exponentially mean-square stable if there exist positive definite matrices $P, Q_i (i = 1, 2), R_j (j = 1, 2, 3)$, such that LMI (7) holds.

$$\begin{bmatrix} \Gamma_{11} & * \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} < 0 \quad (7)$$

where

$$\Gamma_{11} = \begin{bmatrix} \Upsilon_{11} & * & * & * & * \\ R_1 & \Upsilon_{22} & * & * & * \\ (PB_1\bar{\Xi}KC)^T & R_2 & -2R_2 & * & * \\ 0 & 0 & R_2 & \Upsilon_{44} & * \\ B_2^T P & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}$$

$$\Gamma_{21} = \begin{bmatrix} \mathcal{R}A & 0 & \mathcal{R}B_1\bar{\Xi}KC & 0 & \mathcal{R}B_2 \\ 0 & 0 & \begin{bmatrix} \sigma_1\mathcal{R}B_1L_1KC \\ \vdots \\ \sigma_m\mathcal{R}B_1L_mKC \end{bmatrix} & 0 & 0 \\ C_2 & 0 & 0 & 0 & D_2 \end{bmatrix}$$

$$\begin{aligned} \Gamma_{22} &= diag\{-\mathcal{R}, diag\{-\mathcal{R}, \cdots, -\mathcal{R}\}, -I\} \\ \Upsilon_{11} &= PA + A^T P + Q_1 + Q_2 - R_1 \\ \Upsilon_{22} &= -Q_1 - R_1 - R_2 \\ \Upsilon_{44} &= -Q_2 - R_2 \\ \mathcal{R} &= \tau_1^2 R_1 + (\tau_2 - \tau_1)^2 R_2 \end{aligned}$$

Proof: Construct a Lyapunov-Krasovskii functional candidate as

$$\begin{aligned} V(x_t) &= x^T(t)Px(t) + \sum_{i=1}^{2}\int_{t-\tau_i}^{t} x^T(s)Q_i x(s)ds \\ &+\tau_1\int_{-\tau_1}^{0}\int_{t+s}^{t} \dot{x}^T(v)R_1\dot{x}(v)dvds \\ &+(\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}\int_{t+s}^{t} \dot{x}^T(v)R_2\dot{x}(v)dvds \end{aligned}$$

From the definition of Ξ in (3), we can easily know

$$\mathscr{E}[B_1(\Xi - \bar{\Xi})KC] = 0 \quad (8)$$

and

$$
\begin{aligned}
& \mathscr{E}\{[B_1(\Xi-\bar{\Xi})KC]^T\mathcal{R}[B_1(\Xi-\bar{\Xi})KC]\} \\
= & \sum_{i=1}^{m}\sigma_i^2 C^T K^T L_i^T B_1^T \mathcal{R} B_1 L_i K C \qquad (9)
\end{aligned}
$$

Using Lemma 1 and the infinitesimal operator (6) for system (4), we have

$$
\begin{aligned}
& \mathcal{L}V(x_t) = \\
& \mathscr{E}\left\{2x^T(t)P[Ax(t)+B_1\bar{\Xi}KCx(t-\tau(t))+B_2\omega(t)]\right. \\
& +\sum_{i=1}^{2}\left\{x^T(t)Q_ix(t)-x^T(t-\tau_i)Q_ix(t-\tau_i)\right\} \\
& +\dot{x}^T(t)\mathcal{R}\dot{x}(t)-\tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds \\
& \left.-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)ds\right\}
\end{aligned}
$$

Now, we set

$$
J(t)=\mathscr{E}\left\{\int_0^t[z^T(s)z(s-\gamma^2\omega^T(t)\omega(t))]ds\right\} \qquad (11)
$$

By $It\hat{o}'s$ formula and using Lemma 1, we derive

$$
\begin{aligned}
J(t) &= \mathscr{E}\left\{\int_0^t[z^T(s)z(s-\gamma^2\omega^T(t)\omega(t))+\mathcal{L}V(x_s)]ds\right\} \\
&- V(x_t) \\
&\le \mathscr{E}\left\{\int_0^t[z^T(s)z(s-\gamma^2\omega^T(t)\omega(t))+\mathcal{L}V(x_s)]ds\right\} \\
&\le \mathscr{E}\left\{\zeta^T(t)[\Gamma_{11}+\Gamma_{21}^T\Gamma_{22}^{-1}\Gamma_{21}]\zeta(t)\right\}
\end{aligned}
$$

where $\zeta(t) = [x^T(t)\ x^T(t-\tau_1)\ x^T(t-\tau(t))\ x^T(t-\tau_2)\ \omega(t)]^T$.

By using Schur Complement, we can concluded (7) is a sufficient condition to guarantee

$$
\mathcal{L}V(x_t)<0 \qquad (12)
$$

Then the proof can be completed.

In the following, we are seeking to design the reliable controller gain K based on Theorem 1.

Theorem 2 For given scalars $\tau_1,\tau_2,\sigma_i,\mu_i(i=1,\cdots,m)$, the closed-loop system (4) is exponentially mean-square stable with considering sensor distortion, if there exist matrices $X>0,\tilde{Q}_i,\tilde{R}_i>0(i=1,2)$ and matrix Y satisfy LMI (13). Furthermore, the reliable controller gain is $K=YX^{-1}C^{-1}$.

$$
\begin{bmatrix}\tilde{\Gamma}_{11} & * \\ \tilde{\Gamma}_{21} & \tilde{\Gamma}_{22}\end{bmatrix}<0 \qquad (13)
$$

where

$$
\tilde{\Gamma}_{11} = \begin{bmatrix}
\tilde{\Upsilon}_{11} & * & * & * & * \\
\tilde{R}_1 & \tilde{\Upsilon}_{22} & * & * & * \\
Y^T\bar{\Xi}^TB_1^T & \tilde{R}_2 & -2\tilde{R}_2 & * & * \\
0 & 0 & \tilde{R}_2 & \tilde{\Upsilon}_{44} & * \\
B_2^T & 0 & 0 & 0 & -\gamma^2 I
\end{bmatrix}
$$

$$
\begin{aligned}
\tilde{\Gamma}_{21} &= \begin{bmatrix}
AX & 0 & B_1\bar{\Xi}Y & 0 & B_2X \\
0 & 0 & \begin{bmatrix}\sigma_1B_1L_1Y \\ \vdots \\ \sigma_mB_1L_mY\end{bmatrix} & 0 & 0 \\
C_2X & 0 & 0 & 0 & D_2
\end{bmatrix} \\
\tilde{\Gamma}_{22} &= diag\{\Lambda_0, diag\{\Lambda_1,\cdots,\Lambda_m\},-I\} \\
\tilde{\Upsilon}_{11} &= AX+XA^T+\tilde{Q}_1+\tilde{Q}_2-\tilde{R}_1 \\
\tilde{\Upsilon}_{22} &= -\tilde{Q}_1-\tilde{R}_1-\tilde{R}_2 \\
\tilde{\Upsilon}_{44} &= -\tilde{Q}_2-\tilde{R}_2 \\
\tilde{\mathcal{R}} &= \tau_1^2\tilde{R}_1+(\tau_2-\tau_1)^2\tilde{R}_2 \\
\Lambda_i &= -2\varepsilon_iX+\varepsilon_i^2\tilde{\mathcal{R}} \quad (i=0,1,\cdots,m)
\end{aligned}
$$

Proof: By Schur complement, the matrix inequality (7) holds if and only if (14)

$$
\begin{bmatrix}\Gamma_{11} & * \\ \bar{\Gamma}_{21} & \bar{\Gamma}_{22}\end{bmatrix}<0 \qquad (14)
$$

where

$$
\bar{\Gamma}_{21} = \begin{bmatrix}
PA & 0 & PB_1\bar{\Xi}KC & 0 & PB_2 \\
0 & 0 & \begin{bmatrix}\sigma_1PB_1L_1KC \\ \vdots \\ \sigma_mPB_1L_mKC\end{bmatrix} & 0 & 0 \\
C_2 & 0 & 0 & 0 & D_2
\end{bmatrix}
$$

$$
\bar{\Gamma}_{22}=diag\{-P\mathcal{R}^{-1}P, diag\{-P\mathcal{R}^{-1}P,\cdots,-P\mathcal{R}^{-1}P\},-I\}
$$

Due to

$$
(\varepsilon_i\mathcal{R}-P)\mathcal{R}^{-1}(\varepsilon_i\mathcal{R}-P)\ge 0 \quad (\varepsilon_i>0)
$$

which gives

$$
-P\mathcal{R}^{-1}P\le -2\varepsilon_iP+\varepsilon_i^2\mathcal{R} \qquad (15)
$$

we have that (14) holds if (16)

$$
\begin{bmatrix}\Gamma_{11} & * \\ \bar{\Gamma}_{21} & \hat{\Gamma}_{22}\end{bmatrix}<0 \qquad (16)
$$

where $\hat{\Gamma}_{22}=diag\{\bar{\Lambda}_0, diag\{\bar{\Lambda}_1,\cdots,\bar{\Lambda}_m\},-I\}$, and $\bar{\Lambda}_i = -2\varepsilon_iP+\varepsilon_i^2\mathcal{R} \quad (i=0,1,\cdots,m)$

Defining $X=P^{-1}$, $\bar{X}=diag\{\underbrace{X\cdots X}_{m}\}$, we apply the congruence transformation $diag\{X,X,X,X,I,X,\bar{X},I\}$ to (16) and set $\tilde{Q}_i=XQ_iX,\tilde{R}_i=XR_iX(i=1,2)$ and $Y=KCX$. The result can be concluded from Theorem 1. This completes the proof.

Remark 3 To avoid using complicated algorithms, we adopt the inequality (15) to let the Matrix inequality (16) be LMI, although there exists a little conservative in the step (16) $\Rightarrow$ (14) comparing with the cone complementary algorithm [16] which is popular in recent control designs. However, we can choose an appropriate parameters ε_i in the criteria.

Remark 4 From Theorem 2, it can be seen that the solvability of LMI (13) depends on the distribution of the actuator fault taking value in an interval. More information are taken into account in our results comparing with the usual fault modeling method mentioned in Section 1.

IV. AN ILLUSTRATIVE EXAMPLE

In this section, a well-studied example is used to illustrate the the effectiveness of the approaches proposed in this paper.

Consider the following time-delay system (1a) with the parameters:

$$A = \begin{bmatrix} 0.5 & 1 \\ 1 & -2 \end{bmatrix}, B1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; B2 = \begin{bmatrix} 0.25 \\ 0.15 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.5 & 0 \\ 0.2 & 0.3 \end{bmatrix}, D2 = \begin{bmatrix} 0.5 \\ 0.6 \end{bmatrix}, C2 = C$$

$\omega(t) = e^{\frac{(t-6)^2}{0.125}}$, and $\tau(t)$ satisfies $0.05 < \tau(t) < 0.6$, and the initial conditions $x(0) = [-1 \;\; 1]^T (t \in [-0.6 \;\; -0.6])$.

Assuming the admissible set of actuator faults are given by $\bar{\Xi} = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$, $\Delta_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.5 \end{bmatrix}$, that is, about half of the first actuator's gain is missing, and there also exists some fluctuations in both of those actuators. According to Theorem 2, we get

$$K = \begin{bmatrix} -3.7494 & -3.7420 \\ -1.2182 & 0.3056 \end{bmatrix} \tag{17}$$

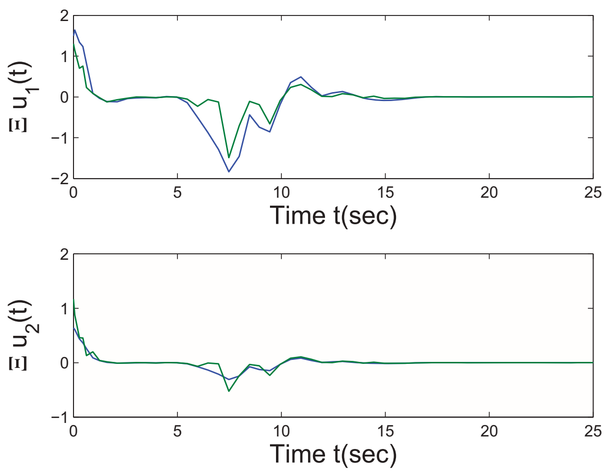

Fig. 1. Inductance of oscillation winding on amorphous magnetic core versus DC bias magnetic field

From Fig.1, we can see that the actuators are suffering stochastic fault. Fig.2 shows the state response and the disturbance $\omega(t)$. It is clear that the controlled plant using the reliable output controller still operates well and maintains an acceptable level of performance.

V. CONCLUSION

In this paper, a new practical actuator fault model is proposed. We concentrate on the reliable control design problem for systems with control input delay, and present a reliable control design methodology to achieve closed-loop stability, not only when the system is operating properly, but also in the presence of certain actuator failures. By means of the feasible positive definite solutions to LMI, A numerical example is given to illustrate the design procedures.

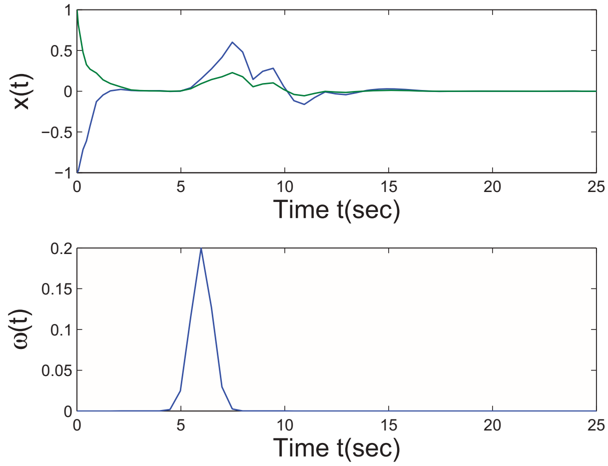

Fig. 2. Inductance of oscillation winding on amorphous magnetic core versus DC bias magnetic field

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REFERENCES

[1] C. De Souza and X. Li, "Delay-dependent robust H_∞ control of uncertain linear state-delayed systems," *Automatica*, vol. 35, no. 7, pp. 1313–1321, 1999.

[2] D. Yue and Q. Han, "Delay-dependent exponential stability of stochastic systems with time-varying delay, nonlinearity, and Markovian switching," *IEEE Transactions on Automatic Control*, vol. 50, no. 2, pp. 217–222, 2005.

[3] E. Fridman and Y. Orlov, "Exponential stability of linear distributed parameter systems with time-varying delays," *Automatica*, vol. 45, no. 1, pp. 194–201, 2009.

[4] M. Vidyasagar and N. Viswanadham, "Reliable stabilization using a multi-controller configuration," *Automatica*, vol. 21, no. 5, pp. 599–602, 1985.

[5] R. Veillette, J. Medanic, and W. Perkins, "Design of reliable control systems," *IEEE Transactions on Automatic Control*, vol. 37, no. 3, pp. 290–304, 1992.

[6] L. Yu, "An LMI approach to reliable guaranteed cost control of discrete-time systems with actuator failure," *Applied Mathematics and Computation*, vol. 162, no. 3, pp. 1325–1331, 2005.

[7] F. Wang, Q. Zhang, and B. Yao, "LMI-based reliable H_∞ filtering with sensor failure," *International Journal of Innovative Computing, Information & Control*, vol. 2, no. 4, pp. 737–749, 2006.

[8] B. Yao and F. Wang, "LMI approach to reliable H_∞ control of linear systems," *Journal of Systems Engineering and Electronics*, vol. 17, no. 2, pp. 381–386, 2006.

[9] F. H. J. HUO Z H, "Fault-Tolerant Control Research for Networked Control System under Communication Constraints," *ACTA AUTOMATICA SINICA*, vol. 32, no. 5, pp. 659–666, 2006.

[10] F. H. J. KONG D M, "Stable Fault-tolerance Control for a Class of Networked Control Systems," *ACTA AUTOMATICA SINICA*, vol. 32, no. 2, pp. 267–273, 2005.

[11] Z. Wang, B. Huang, and K. Burnham, "Stochastic reliable control of a class of uncertain time-delay systems with unknown nonlinearities," *IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMSI: FUNDAMENTAL THEORY AND APPLICATIONS*, vol. 48, no. 5, p. 647, 2001.

[12] Z. Xiang and R. Wang, "Robust L_∞ reliable control for uncertain nonlinear switched systems with time delay," *Applied Mathematics and Computation*, 2009.

[13] H. Wu, H. Zhang, and H. Reliable, "fuzzy control for a class of discrete-time nonlinear systems using multiple fuzzy Lyapunov functions," *IEEE Trans. Circ. Syst. II. v54 i4*, pp. 357–361.
[14] C. Peng, Y. Tian, and E. Tian, "Improved delay-dependent robust stabilization conditions of uncertain T–S fuzzy systems with time-varying delay," *Fuzzy Sets and Systems*, vol. 159, no. 20, pp. 2713–2729, 2008.
[15] X. Mao, "Exponential stability of stochastic delay interval systems with Markovian switching," *IEEE Transactions on Automatic Control*, vol. 47, no. 10, pp. 1604–1612, 2002.
[16] L. El Ghaoui, F. Oustry, and M. AitRami, "A cone complementarity linearization algorithm for staticoutput-feedback and related problems," *IEEE Transactions on Automatic Control*, vol. 42, no. 8, pp. 1171–1176, 1997.